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A user's guide: An equivariant tensor product on Mackey functors

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3. Story of the development

There are two phases to the development of a mathematical paper: the research phase and the writing phase. We sit and think during the research phase. We create pages of notes that mostly consist of wrong turns, doodles, and teardrops, but eventually the light bulb turns on and we develop new mathematics. We have conceived pages of definitions, theorems and proofs and are ready to share our findings with the mathematical community. Hence, we need to turn our notes into a cohesive and publishable document. We move into the writing phase. While the writing phase often becomes overshadowed by the research phase, I believe that developing a coherent story remains a critical process in mathematics research. Why discover new mathematical concepts but then obscure them beneath murky writing? As a result, here I would like to focus on the writing portion of this project.

I grew as a mathematician during the research phase of this project, but I *grew up* as a mathematician during the writing phase. I started to learn how to communicate to the math world, and I developed a deeper understanding of this project and its role in mathematics. This paper summarizes my graduate work at the University of Virginia. As a PhD candidate, I spent a lot of time tinkering with Mackey functors and Tambara functors, and working through examples and confusion. By early 2011 I established a cycle in which I would think for a week, ask my advisor questions, and then spend the next week understanding his answers and formulating new questions. This process continued for a few years, but eventually I made significant mathematical strides with Mackey functors and Tambara functors. I decided to graduate in May 2013. So, in the spring of 2013, with graduation looming, I needed to prepare a thesis for my defense. The writing phase began.

I did not write much during graduate school, and my thesis was the first formal mathematical document I created. As I wrote it I did not think about how to present my research. Instead I just wrote down absolutely everything that

I discovered or learned about Mackey functors and Tambara functors. Moreover, I did not organize it in a logical way and did not develop proper mathematical notation for the story. For example, I devoted ten pages to the proof of what is now Theorem 4.7 in [**Maz16**] because I did not develop a practical way to describe generators and Weyl actions. As a result, these pages consisted of long strings of generators and relations that were impossible to read. (For instance, one page only contained 18 words, and I filled the rest of the page with symbols.) Regardless, I successfully defended my thesis in April 2013. However, while I earned my PhD, I felt dissatisfied with my thesis because I knew I could write a better story. Indeed, I had a significant amount of work ahead of me to turn the thesis into a publishable paper. However, I also needed to prepare for a visiting assistant professor position with a high teaching load. Thus began the three-year process of converting my thesis into a paper.

I worked on this paper on and off between the fall of 2013 and the fall of 2015. I started by developing a language and symbol system for telling my story in a reasonable number of pages. In particular, a crucial part of $N_H^G \underline{M}$ consists of large cartesian products and tensor products of $\underline{M}(G/H)$. I needed to carefully describe how each Weyl group acted on individual elements of these products. Hence, I developed notation that succinctly describes specific elements and how a generator of a Weyl group acts on these elements.

I also found a few mathematical glitches that I needed to fix. For example, I originally ignored the underlying G-action on $i_H^* \underline{M}$ when using $N_H^G i_H^* \underline{M}$ to define a G-symmetric monoidal structure on the category of Mackey functors. This was a mistake, and I had to figure out how this action fit into the story. I also spent a month in the summer of 2015 developing new and improved proofs of the formulas for the norm of a sum and the norm of a transfer in a Tambara functor (Theorems 2.3 and 2.4 in [**Maz16**]). Most frustratingly, I learned that I had no idea how to write a cohesive and coherent mathematical paper. So I turned to the Internet. I googled "how to write a mathematical paper" and read many research papers, paying close attention to their wording and style. Through trial-and-error and emulation of the papers I read, I created a much-improved version of my paper and submitted it to a journal in August 2015.

Then on December 4, 2015 the second best thing for this paper happened: it was rejected. The editor said that the paper should be published but was not strong enough for the journal. I felt incredibly disappointed, but after the tears subsided I resolved to try again. I decided to fix the issues that the referee noted and resubmit the paper to another journal. While working on the revisions, I gave a seminar talk at a nearby university. Here I met a topologist who agreed to read my paper and help me polish it up. This was the best thing to happen to my paper, and acts as a testament to the importance of verbal communication in the mathematical community. This topologist made me realize that my paper did not "tell a story." In particular, I organized the theorems and topics of the paper temporally, but this order lacked coherence to the reader. The topologist also suggested that I focus more on the norm functors that make my research

unique and less on the creation of a G-symmetric monoidal structure that I originally stated as the Main Theorem. Moreover, I realized that I defined terms like the box product and G-commutative monoids way before I needed to use them, so readers did not immediately understand their importance to my project and did not remember what they were when these terms appeared later on. I also needed to break many of my proofs into several lemmas in order to frame a more readable paper. Overall, the structural organization of the paper made it difficult to distinguish between the story and the technical details.

I spent the spring of 2016 completely reorganizing the paper. First and foremost, I rearranged the paper into the following four sections:

- Tambara functors satisfy Tambara reciprocity
- Constructing the norm functors
- The norm functors satisfy the required properties
- Tambara functors are the G-commutative monoids.

With this new outline in mind I developed a main theorem that focuses on the construction of the norm functors, moved definitions to right before I actually use them, and broke long proofs up into digestible lemmas. I also rearranged content within each section and added more intuitive explanations to some confusing concepts.

Now that the re-organization is complete, I have submitted the paper to another journal. Although this process has been long and frustrating at times, through it I have grown as a writer and a mathematician. I learned how to fully develop a story from mathematical research, and I learned the importance of the writing phase in the evolution of a research project. Most importantly, I have created a paper that I am proud to share with my peers.

References

[Maz16] Kristen Mazur, *An equivariant tensor product on Mackey functors*, 2016. arXiv:1508.04062.

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