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A user's guide: Relative Thom spectra via operadic Kan extensions

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4. Colloquial summary

In the long run, the specific result of [**Bea17**] is nice, but it's not the thing that gets me excited about algebraic topology. Rather, it's a small piece of what I consider to be a beautiful story. I described it a little in earlier sections; it's the story of *chromatic homotopy theory.* I won't go into detail about it again, but the main idea is this: when we start to study things like spheres and shapes in really high dimensions, we start to see patterns arise that are coming from number theory. In other words, structures that show up when one deeply studies prime numbers *also* show up when one deeply studies blobs in space.

To me, this is the really beautiful thing about mathematics: mathematicians are blindly groping in the dark, searching for structure, and every once in a while they find the exact same beautiful structure in two different places at once. It'd be like telling one engineer to build a plane, and another engineer to build a superconductor and having them separately build the exact same thing, i.e. an object that is simultaneously a plane and a superconductor. (I don't actually know what a superconductor is, so sorry if this metaphor doesn't make any sense). Of course, we know this could never happen, since these two objects serve such different purposes, and what's more, we understand their purposes relatively well.

However, when it comes to understanding math, we barely have any idea what we're doing. We just plow forward, mostly blind, and sometimes with a vague road map. Sometimes, like in chromatic homotopy theory, we can find connections between two *a priori* disconnected things, but we still struggle to come close to understanding *why* this is happening. Understanding *why* would require not just knowing that I and another blind mathematician happened upon the same spot at the same time, but being able to zoom out and see how that spot relates to all the other spots. And usually we can't do this.

This paper, and much of my work in general, is an expression of my desire to zoom out and see the whole structure. To extend our metaphor a little further, I want to trace the path of each mole and show that secretly, they were taking the same path all along. In other words, the really beautiful theorems to me are the ones that say "Of course you're seeing structure S in topic A and topic B, because topic B is actually just a sub-topic of A and we never knew it!" I am of course biased, so my ultimate goal would be to say that the number theoretic structure we're seeing in chromatic homotopy theory is arising because number theory (in particular arithmetic and algebraic geometry) is itself actually a subfield of algebraic topology. Further, I think many mathematicians would like to show that *many* topics of mathematics, e.g. combinatorics, algebraic geometry, topology, are all secretly different facets of the same crystal.

The reader familiar with physics will probably have heard the term "grand unified theory" before. The idea is that there are theories describing different physical phenomena, like quantum mechanics, and relativity, but those theories are not mutually compatible. Physicists would like for there to be a single framework that encompasses quantum mechanics, classical mechanics, relativity, and every other physical theory we know of. The ultimate goal of my mathematical research can be thought of as the same type of thing. I would love to find a since source from which the similar structures of algebraic topology and algebraic number theory both spring.

References

[Bea17] Jonathan Beardsley, *Relative Thom Spectra Via Operadic Kan Extensions*, Algebraic and Geometric Topology **17** (2017), no. 2, 1151-1162.

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